Discrete Optimisation
Exercise Session 3: Modelling

30th September 2016

After this session, you should be able to solve all exercises of Sections 2.2, 2.3, and 2.4 in the exercises book.

1 Boolean algebra

Exercise 1 (NOT — 2.2.1)**.** How to model a Boolean NOT, i.e. $\neg a$ where a is a binary variable?

Exercise 2 (OR — 2.2.2 (§1))**.** How to model a binary Boolean OR, i.e. the relationship $a \vee b$ between two binary variables a and b?

Exercise 3 (AND — 2.2.3 (§1))**.** How to model a binary Boolean AND, i.e. the relationship $a \wedge b$ between two binary variables a and b?

Exercise 4 (implication — 2.2.4)**.** How to model the fact than an implication $a \Rightarrow b$ must be true, where a and b are binary variables?

Exercise 5 (if-then — 2.2.5)**.** How to model the following expression, where $x \in [0, 100]$ and $y \in \mathcal{B}$?

$$\text{if } y = 1, \text{ then } x = 100$$

2 Thresholds

Exercise 6 (binary activation — 2.3.1)**.** How to model the constraint that the variable $x \in \mathbb{R}^+$ can take a nonzero variable if and only if the variable $y \in \mathcal{B}$ is 1?

Exercise 7 (semicontinous variable — 2.3.2)**.** How to model the constraint that the variable $x \in \mathbb{R}^+$ can take a value in the set $[m, M]$ if and only if the variable $y \in \mathcal{B}$ is 1?

Exercise 8 (threshold detection — 2.3.3)**.** How to model the following expression, where $x \in \mathcal{B}$ and $y \in [0, 100]$?

$$x = \begin{cases} 1 & \text{if } y \geq 50 \\ 0 & \text{otherwise} \end{cases}$$

How does your solution behave for $y = 50$?

3 More modelling

Exercise 9 (volleyball teams — 2.4.1)**.** Make teams for a volleyball tournament. Each team has exactly six members.

Then, consider the gender of the players: impose that each time has at least two men and two women.

For the next step, add the notion of level for each player; they are noted on a scale of one to four. Each team must have at least one person for three different levels.

Finally, define the objective: teams must be as similar to each other as possible. More precisely, the total "variance" of the level and presence of each gender must be as low as possible (the objective must remain linear).

Exercise 10 (attic problem — 2.4.3 (§1, §2))**.**

1. Your aunt passes away, you and your sister are her only heirs. How to solve inheritance problems? Minimise the unfairness when splitting the heritage (consisting of a list of objects to divide amongst the two of you), computed with various criteria: monetary value, volume, sentimental value. Each of you shall get approximately the same amount for each of these criteria out of the heritage of your late aunt.

2. However, your notary is in a hurry, and would want the actual distribution rather soon. Why is your model so slow to solve with many objects? Find an explanation that your notary could understand.
   Hint: what is the value for the first LP relaxation?